The pall-bearers were: President Charles W. Eliot; Ex-President Thomas Hill, Pastor of the First Parish Church, Portland, Maine; Capt. C. P. Patterson, Superintendent of the United States Coast Survey; Professor J. J. Sylvester, of the Johns Hopkins University; Hon. J. Ingersoll Bowditch; Professor Simon Newcomb, Superintendent of the American *Ephemeris and Nautical Almanac*; Dr. Oliver Wendell Holmes; Professor Joseph Lovering; and Dr. Morrill Wyman. A beautiful and simple service was conducted by the Rev. A. P. Peabody and the Rev. James Freeman Clarke.

In the career of Professor Benjamin Peirce, America has nothing to regret, but that it is now closed; while the American people have much to learn from his long, useful, and honorable life.

REMARKS ON SUBSTITUTION GROUPS.

By G. A. MILLER, Ph. D., Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.

[Continued from May Number.]

Since $a.bbc=acb$ and $bc.ab=abc$ it follows that the result is not always independent of the order in which we perform the operations indicated by two substitutions. In the equation $ab.bc=acb$ we call ab and bc the *factors* and acb the *product* and the process is called *multiplication of substitutions*. The above example shows that the law that the product is independent of the order of the factors does not hold true with respect to the multiplication of substitutions.

The number of substitution groups increases very rapidly as the number of letters increases. During the last few years the work of making complete lists of such groups has been carried through ten letters* but no formula has yet been published by means of which the number of such groups can readily be determined for any number of letters.

If an expression involving a given number of letters is unchanged by applying all the substitutions of a group of the same number of letters to it but is changed by applying any other substitution of the same or a lower number

*The number of groups of ten letters exceed one thousand. Complete lists are found in the Quartly Journal of Mathematics as follows: Cayley: Substitution groups for two, three, four, five, six, seven, and eight letters, vol. 25, pp.71-88, 137-155. Cole: List of substitution groups of nine letters, vol. 26, pp. 372-388. Cole: List of transitive substitution groups of ten and eleven letters, vol. 27, pp. 39-50. Miller: Intransitive groups of ten letters, vol. 27, pp 99-116.

A few errors and omissions with respect to these lists have been noted in late numbers of the Bulletin of the American Mathematical Society. The lists are complete in the sense that an effort is made to give all the possible groups of the given number of letters and fairly accurate results have been attained.

of letters, then the expression is said to belong to the given group. We may take, for example, the expression with four letters

$$ac+bd.$$

We see that no matter what values a, b, c, and d may have this expression cannot change its value for any of the substitutions in

$$\begin{array}{llll} 1 & ab.cd & abcd & ac \\ & ac.bd & adcb & bd \\ & ad.bc & & \end{array}$$

but that it changes its value, in general, for any other substitution of four or a lower number of letters; hence we say the given expression belongs to this group, and, conversely, that the group belongs to this expression. From this it can be seen that substitution groups furnish a means by which we may classify such algebraic expressions and thus study the common properties once for all. It has been proved that every integral expression belongs to some group and that an infinite number of such expressions belong to each group. By studying a group we therefore study some properties common to an infinite number of algebraic expressions and from this it follows that the study of substitution groups is a matter of economy in case familiarity with a large number of expressions is to be attained.

The groups of two and three letters are so simple that they are frequently employed without any explanation of their connection with an extensive science. For example, when a factor of an expression belonging to one of these groups can be found by inspection and belongs to the same group as the expression the form of the other factor is often obtained from the fact that it must belong to the same group. This is explained by employing simple properties of these groups in place of the groups themselves. Even in these cases a knowledge of the theory of substitution groups would contribute much to the clear understanding of the matter on the part of the student and in the more complex cases such a knowledge becomes almost indispensible if a comprehensive knowledge is to be attained. In the factoring known as the solution of equations, substitution groups have since the time of Galois played the most prominent part—serving not only to give a comprehensive view of the entire field but also to extent the knowledge with respect to it.